MODELING OF BEHAVIOR AND INTELLIGENCE *

Analysis of the distribution of functions in an organization

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The article considers the problem of analysis and improvement of organizational structure, using the method of monotone systems [1] to identify the structure of the management function distribution matrix. Sufficient conditions for rational change of these matrices are given.

1. Introduction

The various works on mathematical methods of organizational structure analysis can be divided into two broad groups: studies, which aggregate the matrices of management task interdependence [2] and studies, which aggregate the matrices of manager interdependence [3]. Works of the first category focus on groups of tasks that can be identified as independent "manager functions," without relating these tasks to individual managers. Works of second category isolate groups of strongly interacting individuals without relating them to the specific functions in which they interact. Yet it is difficult to develop specific recommendations for improvement of organizational structure without considering exactly what functions of the individual managers should be changed and how.

In this article, we consider the analysis of organizational structure using information about the distribution of management functions between individual managers and applying the conceptual approach of [4] to the structure of organizations. This approach can be briefly described as follows.

The mangers of a given organization are divided into two special subgroups: the subgroup of coordinators and the subgroup of operators. The first group naturally includes the division managers and the staff managers; the second group includes the so-called specialists. An important characteristic of the first group is that its members perform a large number of management functions and frequently interact among themselves and with other managers. A characteristic feature of the second group is that the tasks of the corresponding managers are highly specific and the level of their interaction among themselves and with other functionaries, even within the same administrative unit, is quite low.

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These subgroups do not necessarily encompass all the officers of an organization. Some officers occupy an intermediate position between coordinators and operators. Their function is to transmit coordinating actions to the operators and to communicate information about the results of special tasks to the coordinators. Moreover, the two subgroups are not necessarily disjoint. This intersection, i.e., the existence of managers acting both as coordinators and as operators, is interpreted in [4] as a stress factor in a functioning organization.

This model of organizational structure has definite empirical support [5]. However, its application to the analysis of specific organizations runs into certain difficulties. The first difficulty is that it requires highly comprehensive and detailed input information. As it follows from [6,7], the main problems are encountered in collecting comprehensive and detailed data about the structure of all the basic interaction types between various managers. Even in those exceptional cases when such information can be collected, the data are often unreliable and incomplete. The second difficulty is associated with the uncertainty of the criteria used to identify the two groups. The third difficulty is associated with the need to analyze large volumes of data (a typical number of managers is 20-100, and a typical number of management functions is 100-1000, i.e., a single table of management function distribution consists of thousands of entries).

In this context, the main objective of this study is to develop a method of analysis, which, first, will provide a formal procedure for the definition of the subgroups of coordinators and operators using only information about the distribution of the management functions (which is much easier to collect [8]) and, second, will be capable of processing large “manager-management function” tables.

We decided on the method of monotone functions [1] as the appropriate method for our purposes. It was applied to develop constructive formal definitions of the coordinator subgroup and the operator subgroup, consistent with the conceptual notions of [4] and based solely on information about the distribution of management functions. As we shall see, the two groups are not simply subsets of individual managers, each having a certain property, but actually organic entities whose composition depends on the properties of all the managers in the organization.

On the basis of these definitions, we pose and solve the problem of introducing rational (in a certain pre-specified sense) changes in the observed distribution of management functions. These changes are used to formulate practical recommendations for improvement of the existing organization.

2. Identification of Coordinators and Operators from the "Manager – Management Function" Table

Let the observed data about the distribution of the management function be arranged in a matrix $\Phi = \|\varphi_{ip}\|_N^M$, where

$$\varphi_{ip} = \begin{cases} 1 \text{ if the manager } i \text{ performs function } p, \\ 0 \text{ otherwise;} \end{cases} \quad (1)$$

N is the number of managers in the given list, M is the number of functions. Denote by Y the entire set of management functions, by y_i the subset $y_i \subset Y$ identified by the i-th row of the matrix Φ, and by $W = \{y_1, ..., y_N\}$ the family of all such subsets $\bigcup_{i=1}^{N} y_i = Y$. We say that subset y_i defines the sphere of competence of the i-th manager in the given set Y of management functions (as distinct from the usual notion of competence, which usually linked with special knowledge, experience, etc.).

Take a subset H of the set of managers. This subset is in one-to-one correspondence with certain subfamily in the set W, which will be denoted by the same symbol. With H we associate two subsets of management functions Y^H and Y_H:

$$Y^H = \bigcap_{i \in H} y_i \,, \quad (2)$$

$$Y_H = \bigcup_{i \in H} y_i \,. \quad (3)$$

Using these subsets, we assign to each i-th manager in group H, $i \in H$, two numbers $\pi_1(i,H)$ and $\pi_2(i,H)$, which determine his "place" in the group:

$$\pi_1(i,H) = \left|y_i - Y^H\right| = \left|y_i - \bigcap_{k \in H} y_k\right| \quad (4)$$

$$\pi_2(i,H) = \left|Y^H - y_i\right| = \left|\bigcup_{k \in H} y_k - y_i\right|. \quad (5)$$

The cardinality $|Y_H|$ of set Y_H is naturally interpreted as a measure of functional diversity of the group H, and the cardinality $|Y^H|$ of the set Y^H is interpreted as a measure of interaction intensity between the members of the group. This interaction is based on the notion that the activity coordinating mechanism for a group of managers is mainly linked with those management functions that are common to these managers. The greater the number of common functions for the group of managers, the more complex is the mechanism for coordinating their joint activity. Conversely, common functions as such are largely dependent on the existence of such mechanism [4]. Using this interpretation, we will call $\pi_1(i,H)$ the measure of specificity of the activity of manager i in the group H. It measures the "closeness" of the sets y_i and Y^H. The greater the number $\pi_1(i,H)$, the fewer functions of manager i are performed also by all other members of H, i.e., the greater is the complementing effect of manager i on the common sphere of activity Y^H of all the managers in group H, which determines their interaction intensity. The number $\pi_2(i,H)$ will be called the incompetence level of manager i in the group H. It measures the "closeness" between the sets y_i and Y_H. The greater the number $\pi_2(i,H)$, the fewer functions from Y_H are performed by the i-th manager, i.e., the smaller is his "substituting" effect on the functional diversity Y_H of the entire group H or, in other words, the greater is the complementing effect of the other members of the group H on his activity. In this interpretation, managers y_1 and y_2 such that

$$\pi_1(y_1,H) = \min_{i \in H} \pi_1(i,H), \tag{6}$$

$$\pi_2(y_2,H) = \min_{i \in H} \pi_2(i,H), \tag{7}$$

may be called coordinating and operating centers of the subset H, respectively.

Define two scalar functions,

$$F_1(H) = \min_{i \in H} \pi_1(i,H), \tag{8}$$

$$F_2(H) = \min_{i \in H} \pi_2(i,H). \tag{9}$$

Using the above interpretation of the function $\pi_1(i,H)$, we note that the function $F_1(H)$ determines to what extent the manager with least specific competence sphere complements the interaction domain Y^H of the members of the group H, i.e., the function $F_1(H)$ characterizes the interaction complement-ability level in the group H. At the same time, the function $\pi_2(i,H)$ determines to what extent the manager with minimal incompetence (maximal competence) in the group can be regarded as fully reflecting the functional diversity Y_H of this group, i.e., $F_2(H)$ characterizes the diversity complement-ability level in the group H.

Then the problem of identifying the coordinator subgroup and the operator subgroup may be formulated in the form of two independent problems: find two subsets of managers G_1 and G_2 such that

$$F_1(G_1) = \max_{H \subseteq W} F_1(H), \quad (10)$$

$$F_2(G_2) = \max_{H \subseteq W} F_2(H). \quad (11)$$

This statement of the problem ensures that coordinators are managers representing groups that strongly differ in their common management functions [1] Y^H, whereas operators are managers representing groups that strongly differ in functional diversity as determined by the set Y_H. [2]

Effective solution of the problems (10) and (11) depends on the important property of monotonicity of the families of functions $\pi_1(i,H)$ and $\pi_2(i,H)$. For any pair $i, j \in H$,

$$\pi_1(i,H) \geq \pi_1(H - j),^3 \quad (12)$$

$$\pi_2(i,H) \geq \pi_2(H - j). \quad (13)$$

Because of this property [1], the algorithm to solve these problems reduces to the construction of a sequence [4] $I = \langle i_1,...,i_N \rangle$ of indexes of the elements in W such that for any k $(1 \leq k \leq N)$, if $H_k = \langle i_k, i_{k+1},...,i_N \rangle$, then

$$\pi(i_k, H_k) = \min_{i \in H_k} \pi(i, H_k). \quad (14)$$

The sought set G is then taken as the set H_m with the highest cardinality, such that

$$\pi(i_m, H_m) \geq \pi(i_s, H_s) \text{ for all } s = \overline{1,N}. \quad (15)$$

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[1] These are the heads of major divisions and staff managers (the chief economist, the chief engineer, etc.), for which the corresponding group consists of subordinate unit managers (for the chief economist, these are heads of the planning and financial departments, personnel and payroll departments, accounting department, etc.). Thus, these are precisely the managers identified as co-ordinators in [4].

[2] These are "groups" of narrow specialists generally not involved in the affairs of one another (large incompetence measures), members of these groups are usually scattered in different divisions and hardly interact with one another and with other managers.

[3] For simplicity $H - j \equiv H \setminus j$.

[4] A separate sequence is constructed for each problem, but when the corresponding notation is applicable to either sequence, the identifying subscript may be omitted, i.e., we write I instead of I_1 or I_2, π instead of π_1 or π_2, G instead of G_1 or G_2, etc.

In other words, each group (coordinators and operators) is identified by successively excluding the elements of i_k with minimal value of the function $\pi(i_k, H_k)$ from among the elements of H_k remaining in step k. The elements, in the order of their exclusion, form the sought sequence I and the corresponding sequence of nested sets $\overline{H}$:

$$\overline{H} = \langle H_1, H_2, \ldots, H_N \rangle, \tag{16}$$

where

$$H_1 = W,\ H_2 = H_1 - i_1, \ldots, H_k = H_{k-1} - i_{k-1}, \ldots, H_N = i_N. \tag{17}$$

In the process we obtain the best value of the function $\pi(i_m, H_m)$, the corresponding best element $g = i_m$, and the best set $G = H_m$:

$$\pi(i_m, H_m) = F(G) = \max_{k \in \overline{1,N}} \pi(i_k, H_k). \tag{18}$$

The fact that G is the largest set satisfying (18), i.e., the set with maximum value of the function $F(H_k)$ first encountered in the sequence $\overline{H}$, the following relations conveniently express it:

$$F(H_k) < F(G),\ \forall H_k \supset G,\ \forall i_k \in W - G, \tag{19}$$

$$F(H_k) \le F(G),\ \forall H_k \supseteq G,\ \forall i_k \in G. \tag{20}$$

3. Procedure for local Improvement of the Distribution of Functions between Managers

Suppose that in addition to the matrix $\boldsymbol{\Phi}$, we also know the subgroups G_1^o and G_2^o of managers who from fundamental considerations should be classified as coordinators and executives. Then the solution of problems (10) and (11) is naturally considered as a procedure that checks the consistency of these prior notions with the information about the distribution of the management functions. The greater the difference between the subgroups G_1 and G_2 identified by solving the problems (10) and (11) and the a priori specified subgroups G_1^o and G_2^o, the poor is this consistency.

We will naturally try to find a transformation of the initial matrix Φ into matrix Φ^o such that the given groups G_1^o and G_2^o maximize the functions $F_1(H)$ and $F_2(H)$ respectively, i.e., the new matrix Φ^o ensures perfect consistency with the prevailing conceptual notions. In this way, we will identify the "component" of inconsistency due to "imperfection" of the initial distribution of management functions between the relevant managers.

It is clear that, without additional restrictions such a transformation always exists. However, it may turn out to be unreliable if the new matrix Φ^o is very different from the original matrix Φ. Therefor, we a priori restrict the search to those transformations of the matrix Φ which, first, ensure that the matrix Φ' is close in a certain sense to Φ and, second, bring us maximally close to the consistent solution. This strategy is implemented in what follows by deriving effectively testable sufficient conditions under which a transformation of the set of functions y_l of a single manager l leads to his inclusion in (or exclusion from) the set G_1 or G_2 that solves the problem (10) or (11), respectively. Such transformations of the matrix Φ, which only change a single row Φ_l, are called local.

From the definition of local transformation, the rows of the matrices Φ and Φ' are related by equalities

$$y'_i = y_i \;\; (\Phi'_i = \Phi_i),\; i = \overline{1,N},\; i \neq l$$

(variables and sets relating to the solution of problems (19), (11) on the matrix Φ' are primed).

There are four types of local transformations of the matrix Φ and correspondingly four groups of sufficient conditions.

To formulate the sought conditions, we will consider in addition to the function $\pi(i,H)$ defined on the set H its extension $\Pi(i,H)$ to the entire set W: [5]

$$\Pi(i,H) = \pi(i,H+i) \text{ for all } i \in W \text{ and } H \subseteq W . \qquad (22)$$

[5] To simplify the notation, we will write $H+i$ for $H \cup \{i\}$ and $H-i$ for $H \setminus \{i\} \equiv H - \{i\}$.

1. Let l be the index of a manager $(1 \le l \le N)$ not included in the coordinator subgroup G_1 by formal solution of the problem (10) who should be included in this subgroup from actual considerations, without changing the place of all the other managers relative to the subgroup G_1.

In the set of management functions define a subset Y_l^+,

$$Y_l^+ = Y - \left(Y^{G_1} + y_l\right). \tag{23}$$

This set specifies the entire possible domain for extending the competence of manager l: extending his competence outside this domain, i.e., within the set $Y - Y_l^+$, clearly does not change the value of $\Pi(l,G_1)$ and cannot alter his position relative to the coordinator group.

We now introduce the number n_l,

$$n_l = \min_{i \in G_1} \pi_1(i,G_1) - \Pi_1(l,G_1) + \left|Y^{G_1}\right| - \left|Y^{G_1+l}\right| = \left|y_g\right| - \left|y_l\right|, \tag{24}$$

which we call the characteristic number. In terms of this number, we state the following theorem.

THEOREM 1. If the characteristic number n_l defined by (24) satisfies the inequalities

$$\left|Y_l^+\right| \ge n_l > 0, \tag{25}$$

then any transformation of the l-th row of the matrix Φ that involves replacing precisely n_l zeros by ones in the columns corresponding to set Y_l^+ generates a matrix Φ' for which the solution of the problem (10) is the set $G'_l = G_l + l$.

2. Let us now consider the case when the coordinator subgroup G_l to be transformed contains a manager l who should be excluded from this group. We define the domain of restricted competence of manager l by introducing the set Y_l^-:

$$Y_l^- = y_i - Y^{G_1}, \tag{26}$$

since the restriction of his competence within $Y - Y_l^-$ does not change the value of the function $\Pi_1(l,G_1)$. The characteristic number n_l is calculated from the formula

$$n_l = \Pi_1(l,G_1) - \min_{i \in G_1 - l} \pi_1(i,G_1) + \left|Y^{G_1-l}\right| - \left|Y^{G_1}\right| + 1. \tag{27}$$

We have the following theorem.

THEOREM 2. If the characteristic number n_l defined by (27) satisfies the inequalities

$$\left|Y_l^-\right| \geq n_l > 0\,, \tag{28}$$

$$n_l < \Pi_1(l,G_1) - \max_{k=1,m-1} \Pi_1(i_k,H_k) + 1\,, \tag{29}$$

$$n_l < \Pi_1(l,G_1) - \max_{k=\lambda+1,N} \Pi_1(i_k,H_k) + 1\,, \tag{30}$$

where λ is the number of the element l in the sequence I_1, then any transformation of the l-th row Φ_l of the matrix Φ which involves, replacing precisely n_l ones with zeros within the set Y_l^- generates a matrix Φ' for which the solution of the problem (10) is the set $G'_l = G_1 - l$.

3. Extending the operator subgroup G_2. Suppose that we seek to obtain $G'_2 = G_2 + l$, i.e., to include a selected element l in the operator subgroup G_2.

In distinction from Theorem 1, the inclusion of manager l in G_2 in the case is achieved by restricting his sphere of competence. We define the restricted domain by the set Y_l^-:

$$Y_l^- = y_l \cap Y_{G_2}\,, \tag{31}$$

and take the characteristic number n_l in the form

$$n_l = \min_{i \in G_2} \pi_2(i,G_2) - \Pi_2(l,G_2) + \left|y_l - Y_{G_2}\right| = \left|y_l - y_g\right|. \tag{32}$$

THEOREM 3. If the characteristic number n_l defined by (32) satisfies the inequalities

$$\left|Y_l^-\right| \geq n_l > 0\,, \tag{33}$$

then any transformation of the l-th row of the matrix Φ which involves replacing precisely n_l ones with zeros within the set Y_l^- generates a matrix Φ' for which the solution of the problem (11) is the set $G'_2 = G_2 + l$.

4. If we now define the set Y_l^+ and the characteristic number n_l by

$$Y_l^+ = Y_{G_2} - y_l\,, \tag{34}$$

$$n_l = \Pi_2(l,G_2) - \min_{i \in G_2 - l} \pi_2(i,G_2) + \left|y_l - Y_{G_2 - l}\right| + 1\,, \tag{35}$$

the theorem on the exclusion of manager l from the set G_2 may be stated as follows.

THEOREM 4. If the characteristic number n_l defined by (35) satisfies the inequalities

$$\left|Y_l^+\right| \geq n_l > 0 \qquad (36)$$

$$n_l < \Pi_2(l,G_2) - \max_{k \in \overline{1,m-1}} \Pi_2(i_k,H_k)+1, \qquad (37)$$

$$n_l < \Pi_2(l,G_2) - \max_{k \in \overline{\lambda+1,N}} \Pi_2(i_k,H_k)+1, \qquad (38)$$

where λ is the number of the element l in the sequence I_2, then any transformation of the l-th row Φ_l of the matrix Φ which involves replacing precisely n_l zeros with ones within set Y_l^+ generates a matrix Φ' for which the solution of the problem (11) is the set $G_2 - l$.

Theorem 1 is proved in the Appendix; Theorems 2-4 are proved along the same lines.

A P P E N D I X

Proof of Theorem 1. Consider a defining sequence [6]

$$I = \langle i_1,...,i_\lambda = l,...,q = i_m,...,i_N \rangle$$

and the corresponding sequence of sets

$$H = \langle H_1,...,H_\lambda ...,G = H_m,...,H_N \rangle .$$

Note that exclusion of an arbitrary element $i_\lambda = l$ from the original set W does not alter the order of the remaining elements $W - l$ during the construction of the defining sequence. Indeed, all $\pi(i,H'_k)$, $k < \lambda$, $i \in H'_k$, where $H'_1 = W - l$, $H'_2 = H_2 - l$, etc., decrease by the same amount

$$\left|Y^{H'_k}\right| - \left|Y^{H_k}\right|,$$

which depends on k but is independent on i. Thus, $\min_{i \in H'_k} \pi(i,H'_k)$ is still attained for $i = i_k$.

It is easily shown using rules (14) and (17) for the construction of the defining sequence that this implies coincidence on the interval $k \in \overline{1,\lambda-1}$ of the sequence I constructed on the set W and of the sequence I' constructed on the set $W' = W - l$. With regard to $k > \lambda$, none of the values $\pi(i,H'_k)$, $k > \lambda$, $i \in H'_k$, $H'_k = H_k$ changes and the order is not broken.

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6 Since we only prove Theorem 1 in the Appendix, the subscript 1 is superfluous and is therefor omitted. Note, however, that here all propositions concerning I, G, H_k, etc., are only valid for the function $\pi_1(i,H)$, contrary to our convention in the body of the text.

It thus follows that adding an element l' to the same set $W-l$ does not alter the order in the defining sequence on $W-l$ either.

The extension of the sphere of competence of manager l

$$y'_l \supset y_l,\ y'_l - y_l \subseteq Y_l^+,\ |y'_l| - |y_l| = n_l$$

may be regarded as exclusion of l from W followed by exclusion in $W-l$ of a new element l'. Clearly, this operation introduces a single change in the original defining sequence: the element $i_\lambda = l$ may move to a new place, while the order of all the remaining elements is preserved. We will show that the element l in the new sequence $I' = \langle f_1,...,f_N \rangle$ is placed in position $m-1$, $j_{m-1} = l$, where m is the number of the first element from the set G in the sequence I. In other words, we will prove that the sequence I' constructed using the transformed matrix Φ' differs from the previous sequence in the following sense:

$$j_k = i_k,\ k = \overline{1,\lambda-1},$$

$$j_k = i_{k+1},\ k = \overline{\lambda,m-2},$$

$$j_{m-1} = i_\lambda = l,$$

$$j_k = i_k,\ k = \overline{m,N}.$$

Indeed, for $k < \lambda$, all $\pi'(i,H_k)$, $i \in H_k$, $i \neq l$ decrease by precisely the same amount [7] $\left|Y'^{H_k}\right| - \left|Y^{H_k}\right| = \left|Y^{H_k} - (l \cap y_{l'})\right| - \left|Y^{H_k}\right|$, which is dependent on k but independent of i, whereas the corresponding values of the function $\pi(l,H_k)$, $k < \lambda$ may only increase. At the same time, from the definition of λ, $i_\lambda = l$, we have [8]

$$\min_{i \in H_k} \pi(i,H_k) < \pi(l,H_k),\ k < \lambda.$$

Thus, for $k < \lambda$

$$\pi'(l,H_k) \geq \pi(l,H_k) > \pi(i_k,H_k) \geq \pi'(j_k,H_k).$$

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7 In fact, under the condition of Theorem 1, this difference is zero, since for $k < m-1$, $H_k \supset G$, and $y' - y_l \subseteq y_l^+ = Y - (Y_G \cup y_l)$. Although this is not essential for the proof of Theorem 1, it substantially relaxes one of the conditions in Theorem 2.

8 It will became clear from what follows that the relative position of elements with the same minimal value of function $\pi(i,H_k)$ is not essential for the composition of the core that we are trying to isolate; this allows substitution of strict inequality for week inequality.

This means that in the new sequence I' the element l is not selected in the first $\lambda - 1$ steps. In other words, extension of sphere of competence of manager l may only shift his place to the right in the defining sequence. We will now show that the next interval $\overline{\lambda, m-2}$, the element l is not included in the sequence I'.

Denote by $\sigma(H,l)$ the following difference:

$$\sigma(H,L) = \left|Y^H\right| - \left|Y'^{H+l}\right| = \left|Y^H - y'_l\right|.$$

We can directly verify that for an arbitrary $i \in H$, $l \notin H$, $i \neq l$

$$\pi'(i,H+l) = \pi(i,H) + \sigma(H,l).$$

Writing this equity for $H = H_k$, $k = \overline{\lambda+1, m-1}$, and then $H = G$ and minimizing over i, we obtain

$$\min_{i \in H_k} \pi'(i,H_k + l) = \min_{i \in H_k} \pi(i,H) + \sigma(H_k,l),$$
$$\min_{i \in G} \pi'(i,G+l) = \min_{i \in H_k} \pi(i,G) + \sigma(G,l).$$

From the definition of the set G we have

$$\min_{i \in H_k} \pi(i,H_k) < \min_{i \in G} \pi(i,G), \ k = \overline{1, m-1}.$$

Moreover, since $H_k \supset G$, $l \notin H_k$, then

$$\sigma(H_k,l) \leq \sigma(G,l).$$

Adding the last two inequalities, we obtain

$$\min_{i \in H_k} \pi'(i,H_k + l) < \min_{i \in G} \pi'(i,G+l), \ k = \overline{\lambda+1, m-1}.$$

At the same time, it is clear that for any H, $l \in H$, $G \subseteq H$,

$$\Pi'(l,H+l) = \Pi(l,H) + n_l,$$

and in particular for G

$$\Pi'(l,G+l) = \Pi(l,G) + n_l$$

Substituting (24) for n_l in the theorem, we obtain

$$\Pi'(l,G+l) = \left|y_i\right| - \left|Y^{G+l}\right| + \left|y_g\right| - \left|y_i\right| = \left|y_g\right| - \left|Y^{G+l}\right|.$$

Since in Theorem 1, $Y^{G+l} = Y'^{G+l}$, we obtain the equality

$$\Pi'(l,G+l) = \left|y_l\right| = \left|Y^G\right| + \left|Y^G\right| - \left|Y'^{G+l}\right| = \min_{i \in G} \pi(i,G) + \sigma(G,l) = \min_{i \in G} \Pi'(i,G+l). \quad \text{(A)}$$

Thus, for $\lambda < k \le m-1$

$$\Pi'(l,H_k+l) \ge \Pi'(l,G+l) =$$
$$= \min_{i\in G+l} \Pi'(i,G+l) = F'(G+l) > \min_{i\in H_k} \Pi'(i,H_k+l) =$$
$$= \min_{i\in H_k+l} \Pi'(i,H_k+l) =$$
$$= \pi'(j_k,H_k+l) = F'(H_k+l) = F'(H'_{k-1}).$$

This means that in the new sequence I' the element l is not selected in the interval $k = \overline{\lambda,m-2}$. The equality (A) indicates that in the sequence I' the element l is selected in the step $(m-1)$.

We have thus obtained a new defining sequence. It remains to show that the best value of the function $F'(H'_k)$ is attained on the set $H'_{m-1} = G' = G+l$.

To prove this, consider three sections of the sequence $\overline{H}$:

for $k < \lambda$

$$F'(H'_k) = \min_{i\in H_k} \Pi'(i,H_k) \le F(H_k) < F(G) < F(G) + \sigma(G,l) = F'(G+l),$$

for $\lambda \le k < m-1$

$$F'(H'_k) = F'(H_{k+1}+l) < F'(G+l),$$

for $k \ge m-1$

$$F'(H'_k) = F(H_k) \le F(G) \le F'(G+l). \blacksquare$$

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