Irrational Behavior when Buying and Selling Shares on Stock Market:
Monotone Linkage Choice Model *

Abstract. We focused on the possibility of irrational transactions when buying shares at a higher price. To show this phenomenon, we use an innovative stock market statistical analysis procedure where companies with higher stock performance over other companies with lower stock performance have an advantage in predictive power.
JEL: C25; G17
Keywords: stock market; credentials; monotonic; system; kernel

1. INTRODUCTION

It is well-known that irrational behavior can occur in the stock market when buying and selling company shares. This behavior is often driven by emotions such as fear, greed, or panic, which can lead investors to make decisions that are not rational or logical. To analyze this phenomenon, we can use statistical analysis techniques to identify patterns in the buying and selling behavior of investors. One such technique is to identify companies that have a higher dynamics of change compared to other companies. Higher dynamics company shares are often more sensitive to changes in market conditions and can provide a risk-averse insight into the behavior of investors.

For example, the price-to-earnings ratio (P/E ratio) is a commonly used indicator in stock analysis. This ratio compares a company's current stock price to its earnings per share (EPS) and can provide insight into how the market values the company's stock. A high P/E ratio can indicate that the market expects the company to grow rapidly in the future, while a low P/E ratio can indicate that the market does not have high expectations for the company. However, the P/E ratio is not always a reliable indicator of a company's future performance. Investors may become overly optimistic or pessimistic about a company's prospects, leading them to buy or sell the stock at a price that does not reflect its true value. This can result in a bubble or a crash in the stock market.

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* Münich Personal RePEc Archive, https://mpra.ub.uni-muenchen.de/101591/ (Acessed 23.12.2021)

To avoid these pitfalls, investors should focus on fundamental analysis, which involves studying a company's financial statements and business operations. By understanding the company's strengths and weaknesses, investors can make more informed decisions about when to buy or sell the stock. Additionally, investors should avoid making decisions based solely on emotions and should instead rely on rational analysis and objective data.

Let's start the analysis with a "visual" or "pedagogical exhibit". When accepting the order in a restaurant, the sommelier informs the guest that some of the preferred choices are unavailable, what can lead to irrational behavior on the part of the guest or the sommelier. The guest may be more willing to try cheaper wines that he or she initially overlooked, or the sommelier may suggest more expensive wines even though there are cheaper and equally good options available. Indeed, the absence on the list of the most expensive wines, for sure, will encourage guests to expand the list of cheap wines or at least keep the choice. On the contrary, the lack of approved, at first glance, cheap wines may induce the sommelier to suggest more expensive wines in favor of others available for order, also cheaper, but quite good and better wines. More often than not, guests agree with such a proposal. This irrational behavior was our main motive for informing the reader about "these events." The phenomenon of irrationality is discussed from the point of view of an innovative statistical procedure and illustrated on the basis of a probabilistic-statistical analysis of the numerical indicators of the exchange market.

In probabilistic-statistical analysis two opposing approaches can be distinguished: from subjective to objective knowledge and in the opposite direction — from objective to subjective. In the first approach, such specialists as a physician, biologist, astronomer, practitioner or market analyzer... those who have knowledge in their field, use data visualization for objective statements about the obtained estimates of experimental data, observations, etc. With this approach, from a subjective assessment to an objective assessment, probability research includes: Markov processes (Rogers and Williams, 2000, [1]), Lévy processes (Applebaum, 2004, [2]), Gaussian processes (Lifshits, 2012, [3]), random fields (Adler, 2010, [4])… Statatistical analysis includes space state models (SSM, Koller and Friedman, 2009, [5]), parameter estimation (Walter and Pronzato, 1997, [6]), management and decision-making problems (Narula and Weistroffer, 1989, [7]), continuous modeling, multiple time series (Voelkl et al, 2012, [8]) and computational methods (Mirkin et al, 1995, [9]). In both areas the knowledge of the distribution of judgments about the object under study is necessary that is not always the case.

The objective to subjective assessment (Frey and Võhandu, 1966, [10]), both to statistical and probabilistic indicators, at first glance, seems to be contradictory. It seems that specialized knowledge is also required. Nevertheless, it is very possible to do without special knowledge, as well as knowledge about the distribution of numerical parameters — indicators.

The procedure of the objective to subjective approach considered below could be called the "blind glance of statistical scoring", which is what we need. The only thing the Data Explorer uses in blind scoring is that one number is

greater/less than another. If common sense is achieved, then the well-known law of parsimony or **"Occam's razor"** will come into force. A procedure that requires fewer assumptions about reality can be considered the most reliable. Our parsimonious procedure is somewhat consistent with the postulates of bounded rationality of choice in buying and selling shares in the stock market

In particular, when studying the dynamics of time series, instead of absolute values, we are often interested in indicators over/under estimating these values relative to some given threshold u. Since the procedure described in the article can set the threshold of significance u of absolute values in a certain time interval, now the same procedure can be applied to the "induced" indicators $\pm\Delta$ of "overshoot" or "undershoot". As a result, we will be able to calculate some interval $\left[-\Delta_1+u, u+\Delta_2\right]$, where $-\Delta_1$ is the underestimation undershoot, and $+\Delta_2$ is the overestimation overshoot threshold relative to the same absolute threshold u. Download an EXCEL spreadsheet (see below), where the Ctrl+s macro can be applied to both positive and negative values in columns, rows, or even tables.

In summary, the findings and experiments conducted using an Excel spreadsheet and share prices on the Nord-net stock market did not confirm the postulates of independence of rejected alternatives nor the succession or strict succession with the most dynamical shares in the market. Investors who are more risk-averse may find these insights particularly valuable as they can use them to make more informed decisions about when to buy or sell stocks, or to adjust their investment strategies in response to changing market conditions. This suggests that transactions to buy more dynamical shares may be irrational within the framework of our monotonic choice model. So it's important to always approach investing with caution and to do your own research before making any investment decisions. Stocks with higher dynamics can still be subject to irrational behavior and unpredictable price movements,

2. PARSIMONIOUS APPROACH

The wine list is ordered in descending order of price, and **1** multiplies the price of the most expensive wine, **2** multiplies the next local price, then **3** the next, and so on. We call these numbers as price credentials. The local maximum of credentials and the price of wine are selected when this peak location from the top of the ordered list where the maximum is reached. The guest decides to accept the price of the wine at the local credential maximum as an acceptable level of price significance when choosing wines with a higher or equal price level, e.g., the list $\mathbf{10^2, 9^2, 8^2, 7^2, 6^2, 5^2}$, … suggests that the peak of this sequence is located at $\mathbf{7^2=49}$.

Before we get to the main part of the note, we will highlight some of the points of our short notice. We use extended quotes from well-known experts on the decision theory (bounded rationality) that was found online (Tomasz Strzalecki,TS, Accessed via Google: December 28, 2021, available to download from http://datalaundering.com/download/notes.pdf), since excessive or our own repetition of the main provisions of the theory, as it seems to us, cannot improve the explanation of the postulates of the theory.

3. SIGNIFICANCE INDICATORS

Here we will only look at some of the details of our procedure for analyzing stock market data. Let's define a set of shares $p_j \in W$, $|W| = n$ of n companies, $j = \overline{1,n}$. In particular, suppose that in the sample denoted by the letter H, all potential candidates are presented as companies according to the most significant for investments or sales. We can further define a totality of sets $\{H\}$ of all 2^n samples $H \subseteq W$. Let credentials $\pi(p_j, H) = p_j \cdot |H|$ (in terms of Kempner et al., pp. 19-24, 1997, [11], as monotone linkage functions, [12]) evaluate the level of significance.

The procedure for finding the most significant companies is easy to set up. First, all the shares p_j, are sorted in descending order, constituting (like prices order in wine list) the order $\langle p_j \rangle$, and then a sequence of credentials $\overline{\pi} = \langle \pi_j \rangle = \langle p_j \rangle \cdot j$, $j = \overline{1,n}$, is constructed. The sequence $\overline{\pi}$ is called defining. The list of company shares $\langle p_j \rangle$, in contrast to original list p_j, is necessary descending.

4. SIGNIFICANCE LEVEL

The credentials $\langle \pi_j \rangle$, are single peaked, where the peak denotes the kernel H^* (Mullat, 1971-1995, [13]) of a monotone system. The set H^* constitutes the rational, i.e., the monotone linkage choice implemented in our findings. The $u = \arg\max_{j=\overline{1,n}} \langle \pi_j \rangle$ denotes the peak u at the location k^* from the top of the defining sequence, where the local maximum of $\langle \pi_j \rangle$ is reached. This value u will be called the significance level p_j of a company j, $j = \overline{1,n}$.

Proposition. *Among the totality of all samples* $H \subseteq W$*, i.e., among all the sets* $\{H\}$ *of all* 2^n *samples, the kernel* H^* *guarantees reaching the global maximum of the credential function* $F(H)$ *of samples* H *equal to* $\min_{p_j \in H} \pi(p_j, H)$*:* $H^* = \arg\max_{H \subseteq W} F(H)$.

The proposition confirms the postulate of independence from rejected alternatives, which has been studied in two-person games since 1950 when solving the bargaining problem of John F. Nash [14]. However, it also includes a constraint on higher dynamic shares p_j, if they are taken into account.

5. BOUNDED RATIONALITY POSTULATES

We emphasize it once again, as said, that the theory of bounded rationality choice (e.g., Arrow 1959, [15]) is better explained not by my own words, but by the words of specialists in this field. I downloaded these notes also from the public domain, licensed under a Creative Commons Attribution Non-Commercial-No Derivatives 4.0 license, Tomasz Strzalecki, TS, **available online** (Accessed: 9 July 2020). Below I shell slightly modify the nomenclature for my own purpose.

"These notes are based on lectures I gave at Harvard in 2010-17 and also those I gave when visiting the Cowles Foundation at Yale in 2012…

...Models in Economics are micro founded, which means that at the bottom of every model there is an economic agent (often many of them) choosing an action from an opportunity set. This set, often called the budget set or a menu, represents the various actions that are available to the agent. But which action will the agent choose? By far the most popular theory is that the agent will choose the alternative with the highest utility. In the most basic version of the theory the utility function is an as-if concept, which means that we don't claim it exists in any material sense nor is it necessarily related to the agent's well being, emotions, or biology. It is just a mathematical construct that helps the analyst make sense of observed choices: the agent behaves as if he maximizes a utility function. We don't care how the agent manages to maximize the utility (it may be a hard mathematical problem) because utility is just a language of description and it is not taken literally."...

...Let X *be a set of alternatives that the agent is choosing between. They can be either immediate "payoffs" (for example different candy bars of which your diet lets you eat only one) or more structured objects (such as retirement plans). There are three main "languages" that help us describe choice:"*

The following item is the most important for us.

The choice function (TS): *"...The analyst observes which element the agent chooses from every nonempty set* $A \subseteq X$. *Let* $c(A)$ *denote that element..."*

We use an uppercase letter for the selection function $C(A)$, which emphasizes that the user can observe for analysis not a single element but a set $C(A)$ to decide the set of best alternatives selected from A. In fact, the set A selected for choice action will be considered as an area in the EXCEL spreadsheets — as cells in a column, a row or tables that can be selected using the "pasted" option. The choice function $C(A)$, was programmed in EXCEL, http://datalaundering.com/download/TyskeAktier29062020%202.xls, available online (accessed December 27, 2021), using macro Cntrl+s, which follows the basic wine selection procedure described above in Section 4

Finally, we recall in a more formal form the postulates (cited by Aizerman and Malishevski, 1981, pp. 65-83, English version translated from Russian, p. 189, [16]) that we will note in connection with the procedure of supposedly rational choice of our guests in restaurant:

- *Independence with respect to dropping rejected alternatives (or, for brevity, elimination of options), Postulate 5 (Chernoff, 1954, pp. 422-443,* [17]*) or Axiom 2 (Jamison and Lau, 1973, pp. 901-912,* [18]*):*

$$C(X) \subseteq X' \subseteq X \text{ that } C(X') = C(X);$$

- *Succession, which is the same as Postulate 4 (Chernoff, 1954), or condition* α *(Sen, 1971, pp. 307-317,* [19]*) or the axiom C2 of Arrow-Uzawa (Arrow, 1959, pp. 121-127):*

$$C(X') \supseteq C(X) \cap X'$$

- *Strict Succession or constant residual choice (it is the same as postulate 6 (Chernoff, 1954), and one of the forms of the "weak axiom of revealed preference" of Samuelson, , i.e., the axiom C4 (Arrow, 1959, pp. 121-127):*

$$C(X') = C(X) \cap X'.$$

6. Findings and Experiments

It seems that our experiments did not support the postulate of independence of rejected alternatives on the Nord-net stock market nor the postulate of succession or strict succession, except for the most stable shares. It appears that excluding certain higher dynamic shares from the consideration may encourage buyers to purchase stable shares that are still available for sale instead of more risk-averse ones. This implies that, within the framework of the presented monotonous choice model, transactions for the purchase of more risky shares could be considered irrational. These findings are specific to the framework and context of the presented model and may not necessarily apply to other models or real-world situations. It's advisable to evaluate different models and analyze the data from various perspectives to gain a more comprehensive understanding of the underlying phenomena.

Overall, these findings suggest that investors may be influenced by factors beyond objective market data, such as emotions, biases, or social pressures. Therefore, it is important for investors to be aware of their own biases and to base their decisions on objective data and analysis. Additionally, the findings suggest the importance of diversification and considering a wide range of investment options when making decisions in the stock market.

Appendix

Some comments are necessary in order to explain the implementation of the aforementioned “procedure” for the analysis of the stock market dynamics. The reliability of data on the sale and purchase of shares on the market is guaranteed by the fact that the Nordet online, exchange was used, where all transaction data have been available to everyone. The spreadsheet was compiled using Nordnet public domain https://www.nordnet.dk/markedet/aktiekurser/ (Accessed: Monday, December 27, 2021).

The stock market monitoring spreadsheet consists of 620 Germany companies. Some columns show relative ±% rise and fall in shares prices, last purchase, sale, etc. As you can see, some cells differ from others in certain patterns and frames. These highlighted patterns and frames are the result of using the macro — Cntrl-s. Cntrl-s means, as said, that an analysis of the significance levels of the negative/positive values of the stocks indicators dynamic has been conducted. Using the macro in columns, rows, or selected *("pasted")* areas of the spreadsheet in their entirety may consist of negative/positive numbers distributed throughout the areas without any special order for negative or positive numbers. However, the standard EXCEL data sorting options allow you to sort selected areas in ascending or descending order depending on the specified columns or rows. Thus, having, for example, negative values scattered across a spreadsheet in different cells, these cells can be redistributed together into "contiguous areas" of negative or positive values in the columns or row patterns to satisfy the necessary conditions. Such contiguous areas can help visualize the analysis results.

A note may be helpful. If macro Cntrl-s produces only a few selected cells or an unsatisfactory small number of special patterns and frames — in the case of very high positive or very low negative values — the same macro can be reused, now outside these sharp numerical jumps that were designated as unsatisfactory, i.e., when the macro detects too small areas. In doing so columns or rows must be first reordered in ascending descending order to bring the positive/negative indicators into contiguous areas. Now, reusing Cntrl-s macro in the gap, i.e., against not yet patterned and framed cells, to add additional meaningful cells in addition to previously selected cells, can help to improve the situation.

Negative significance level →	**-1.57%**	**€ 34.60**			
Positive significance level →	**1.36%**	**€ 0.60**	**€ 44.80**	**€ 43.01**	**€ 44.59**
Company in Germany	Today %	Today +/–	Last	Buy	Sell
1&1 DrillischAktiengesellschaft	**-1.65%**	€ 0.38	€ 22.67	€ 22.65	€ 22.70
11 88 0 Solutions AG	**-2.16%**	€ 0.03	€ 1.36	€ 1.36	€ 1.42
2G En. AG	**4.46%**	**€ 2.80**	**€ 65.60**	**€ 65.60**	**€ 66.40**
3M Co.	0.43%	€ 0.58	**€ 136.08**	**€ 134.86**	**€ 136.08**
3U Hold. AG	**2.56%**	€ 0.04	€ 1.60	€ 1.58	€ 1.64
4 SC AG	-0.60%	€ 0.01	€ 1.67	€ 1.63	€ 1.67
4basebio AG	**-1.97%**	€ 0.04	€ 1.99	€ 1.99	€ 2.00
.....	.....	.....	.....	.....	.....

Stock Market Table, shortened.

One can read the following about **Nordnet** (https://nordnetab.com/about/nordnet-overview . Accessed: 23 August 2020):

> **"***Our target group is Nordic savers and investors. We offer products and services to both experienced investors and beginners, no matter if they have knowledge or need guidance, wish to spend hours on your investments every day or simply review your savings a few minutes a week."*

Our vision is to become the Nordic private savers' first choice. To achieve this objective, we must always continue to challenge and innovate, keeping user-friendliness and savings benefit at the top of the agenda. Only then can we achieve the high level of customer satisfaction and brand strength required to become a leader in the Nordic region in terms of attracting new customers and producing loyal ambassadors for Nordnet.

We help people and money grow. We are passionate about creating a world-class user experience. We simplify to make it easier for people to make smart investment decisions. We share our knowledge and inspiration without any hidden agendas.

The overarching purpose of Nordnet's operations is to democratize savings and investments. By that, we mean giving private savers access to the same information and tools as professional investors. This purpose has driven us since we started in 1996 and remains our direction to this day. In the 1990s, the idea of democratization entailed offering easily accessible and inexpensive share trading via Internet, and building a fund supermarket with products from a number of different companies where savers could easily compare returns, risk and fees. During the journey, we have simplified matters and pressed down fees on, for example, pension savings, index funds and private banking services. In recent years, we have democratized the financial sector with, for example, the stock lending program. We are always on the savers' side, and pursue issues of, for example, the right to transfer pension savings free of charge and reasonable and predictable taxation of holdings of stocks and mutual funds.

Our target group is Nordic savers and investors. We offer products and services to both experienced investors and beginners, no matter if they have knowledge or need guidance, wish to spend hours on your investments every day or simply review your savings a few minutes a week."

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